

Nr. 5 / Lemaître's Nebula, or Primeval Atom ?

From Newton to 5-Dimensionality, Black Hole Cosmology, Hamilton-Jacobi, One Robertson-Walker Universe (Part 1)

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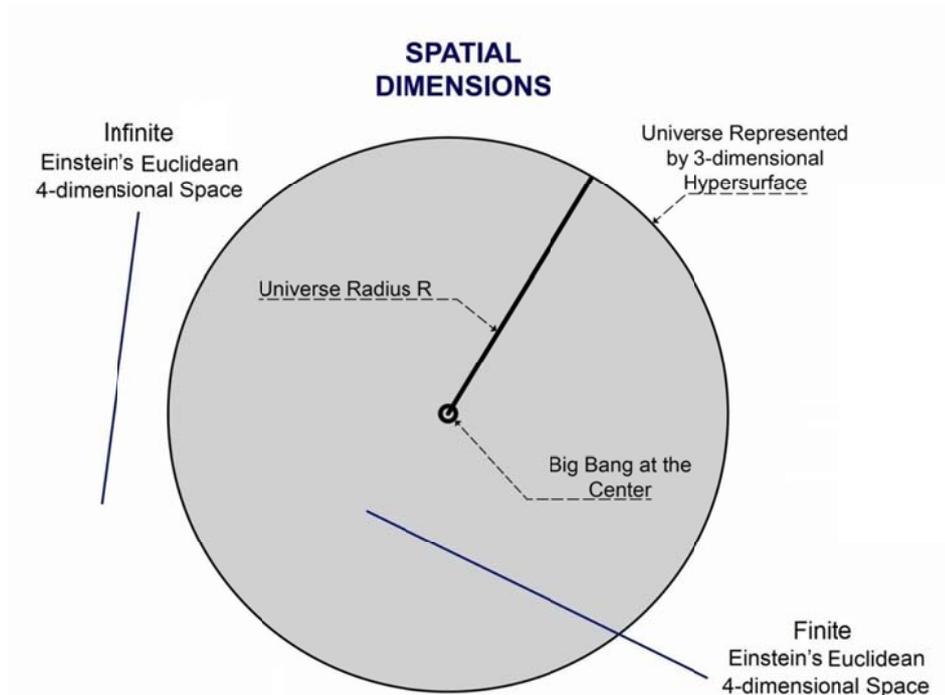


Figure A

Abstract

Contrasting with dark energy cosmology seemingly viewing the universe as a star whose gravitation exclusively acts on its own source, a two-body configuration leads to Schwarzschild's cosmological time coupled with the negative source mass of a primeval black hole. To the end, we follow Einstein's geometric approach in his first (static) universe, hereafter referred to as *Einstein's universe* (anterior to that with cosmological constant in 1917), whose 3-space metric in standard form is structurally identical to that in the spherical Robertson-Walker metric.

Key-Words: Lemaître's primeval atom, Einstein's universe, Vacuum energy, Higgs, Tonnelat, Hamilton-Jacobi, Vinti, Dirac, Repulsive gravity, Double big bang

I-A Overview

In 1927, Georges Lemaître discovered the expansion of the universe [1-A], confirmed by Edwin Hubble in 1929. To simplify the particulars about the independent works on spacetime metrics by Friedmann, Robertson and Walker, what follows uses the Robertson-Walker metric, hereafter referred to as R-W metric.

1. This paper intends to uncover Einstein's reasoning and the implications of his first static universe **[1-B]** according to general relativity formalized by him in November 1915. Einstein focused on the dimensions of space to construct a 3-dimensional metric, to which he added the source-free term $c^2 dt^2$ from special relativity, because the unique source of gravitation was the mass of the universe itself (no exterior source). Admissible for a non expanding universe, this situation did last until now, in spite of the search of new force(s) pushing for expansion, possible dark energy, etc.

Nevertheless, three spatial dimensions were insufficient to close theoretically Einstein's first universe, so that a 4th dimension of space was imperative (calculations in Section V): What follows details Einstein's footnote in which he referred to a Euclidean 4-dimensional space as an *artifice*, because he used it only once. Moreover, Einstein, who just finished 4-dimensional general relativity, could hardly take seriously four dimension of space, to become five with the time, so that nobody would *swallow* five dimensions in general relativity already put in doubt by the scientific community in Einstein's time, situation lasting until 1965, eventually more (?), in European universities (personal remembering).

In some contrast, the 4th space dimension x^4 only figures in next equations (10) to (16), its explicit presence being eliminated in the final equations, so that Einstein's universe looks 4-dimensional, as current general relativity. However, this dimension x^4 remains implicit in all possible cosmological models, according to Figure A showing all values of the universe radius R , from the big bang ($R = 0$) to the today R value. This universe radius R is time-dependent, as well as the expanding universe 3-hypersurface, but both cannot be interpreted as kind of imprecise time measures, because the time-dependent circumference exclusively represents the universe 3-hypersurface, which did expand since the time zero until now. Importantly, this circumference also expanded over the whole *Finite Euclidean 4-dimensional Space* in Fig. A (5-dimensional when including time).

Since explaining the expansion of the universe, the R-W metric was considered incomparably superior to what Einstein did in his seemingly primitive universe. However, the R-W metric was theoretically derived by Einstein in 1916 **[1-B]** (roughly eleven years before Lemaître), as evidenced in Section V, the only change required for theorizing an expanding universe being nothing more than making Einstein's universe radius *time-dependent* (one word difference). Moreover, Einstein's procedure appears today indispensable to improve the outdated R-W metric, exemplified in Section VI.

2, We foresee the possibility of repulsive gravity (see further) originating in the negative mass of a primeval black hole, which here constitutes the unique initial condition. When emitting subsequently a definite quantity of positive energy to build a spacetime with a universe, the corresponding amount of black hole negative energy will raise of the same absolute quantity (negative number of energy units), by virtue of strict mass-energy conservation. To produce the universe with its spacetime of constant total mass M , this operation can be executed once, avoiding continuous, or sequential, energy "creation" perturbing the universe equilibrium, for example with small big bangs, etc. Somewhat summarizing, all this is about the universe and its spacetime originating from less than nothing, in reference to the negativity of primeval black hole mass.

The problem of universe expansion is mainly mechanical because the universe is essentially an electrically neutral, massive, object, hardly anything else for now. This justifies the use, in Section IV, of Jacobi's mechanics, whose Hamiltonian energy covers Hamilton's contribution. This somewhat differs from the Hamilton-Jacobi equation, basis of celestial-orbital mechanics, whose more abstract character is less intuitive when looking for something a bit novel. Using this method quickly evidenced the negative mass of the primeval black hole necessarily preceding the apparition of the universe, this negativity being used to complete the R-W metric in Section VI. According to this, billions years ago, this black hole formed our universe through ejection achieved by repulsive gravitation caused by the negative black hole mass acting gravitationally on our universe, just formed with a positive mass. As a result, this giant black hole is still located outside our universe, at the exact center of Figure A. We will never observe it in a naive fashion, but this black hole had all reasons to exist and still does today.

I-B Introduction

During the last fifty years or so, energy density estimates of the universe varied between 1.4 % [2] to 2.5%, even 5 % (?), reason why some Internet comments qualify cosmology as *erratic*, One also reads *spacetime geometry is influenced by whatever matter and radiation are present* (General Relativity, Wikipedia). Both views do not look very compatible although the Wikipedia vision seems hard to dismiss. This situation reflects the difficulty to conciliate mathematical viewpoints with practical concerns, besides accepting that usual metrics are problematic in cosmology.

However, a legitimate mathematical perspective would mean little if limiting mass-energy to act exclusively on itself, neglecting other sources of gravitation when all sources should be accounted for. In what follows, the radial expansion of the universe corresponds to a two-body system, whose center accommodates a black hole, according to Schwarzschild's solutions, (gravity is static here). By virtue of (static) centered-spherical symmetry, this central black hole remained at the exact place of the big bang that originated our universe. Besides, gravitational energy, not being explicit in general relativity, impedes the intuition to work as in technology, situation solved by Newton's potential in Jacobi's formulation depicting cosmology as a typical exterior case implying an adaptation of the R-W metric.

Physical-Mathematical Conventions and Dimensionality

This paper agrees with Einstein's choice of a closed, spherical universe [1-B].

Changing the sign of Riemann's tensor from refs. [1-B] and [2], Einstein's equation for gravitation reads in four dimensions

$$R_{ab} - (1/2)g_{ab} \cdot R = K \cdot T_{ab} \quad (A)$$

where Einstein's constant of gravitation K is defined by

$$K = 8\pi G/c^4 \quad (B)$$

with G being Newton's gravitational constant. The energy tensor of a perfect cosmic fluid with pressure then reads

$$T_{ab} = (\sigma_0 c^2 + p)w_a w_b + p \cdot g_{ab} + \lambda \cdot g_{ab} \quad (C)$$

($w^a \equiv dx^a/ds$), where σ_0 is the rest mass density of matter, p is the usual pressure.

The scalar field λ represents the positive energy density of vacuum, expected to be R -dependent and related to the Englert-Higgs field in vacuum.

In conformity with current terminology, the word *dimension* most often refers to spatial dimensionality. For example, a 3-dimensional hypersurface is in reality 4-dimensional when including time. Einstein's finite 4-dimensional Euclidean space in Fig. A is 5-dimensional, due to the presence of time in the metric, etc. According to details above, the infinite Einstein Euclidean 4-space might simply be replaced by nothingness since apparently playing no role in the expansion of the universe.

The organization of this paper is the following. Section II is about the interior and exterior cases in cosmology. Section III details some generalities. Section IV introduces the relation between cosmic gravitation and the Hamilton-Jacobi formalism. Section V recalls the origin of the Robertson-Walker metric in Einstein's universe. Section VI proposes a variant of the R-W metric.

II. Interior and Exterior Cases in Cosmology

In unifying attempts during the 1919-1955 period [3], the interior case refers to the equations in matter (sources and particles of finite size), the exterior case being that of unified field(s) outside sources and particles. At the micro level, Tonnelat's comments evidence the dichotomy between two distinct field structures, inside and outside a massive (charged) elementary particle [4].

The galactic case is not detailed here (Part 2). At an intermediate level constituted by galactic and intergalactic configurations, both structures coexist on a same footing. In cosmology, what follows is about the formal separation between interior and exterior cases.

1. The Interior Case

The interior case, without unique center since isotropic, is that viewed by an observer inside the universe in free fall [2]. This observer differs formally from an outside observer such as a cosmologist characterizing the distinct exterior case. One represents the interior case by the isotropic 3-dimensional hypersurface (three space dimensions) constituting the flat geometric background of locally vanishing cosmic gravitation [1-B], this hypersurface being 4-dimensional when including time. Einstein's word *Euclidean* thus means $A = B = 1$ in the 4-dimensional metric

$$ds^2 = B(c^2 dt^2) - A[dr^2 + r^2(d\theta^2 + \sin^2\theta \cdot d\phi^2)] \quad (1)$$

written in spherical coordinates in special relativity. This expresses the essence of general relativity based on the equivalence principle (Einstein [1-B]).

2. The Exterior Case Defining Cosmology

In Fig. A, Einstein's Euclidean 4-space appears inside and outside the circumference representing the 3-hypersurface of zero thickness, according to the words *A spherical manifold of three dimensions, embedded in a Euclidean continuum of four dimensions* [1-B]. However, Einstein's footnote *The aid of a fourth dimension has naturally no significance except that of a mathematical artifice* evidences that Einstein underrated higher dimensionality by referring to it as an artifice, which is understandable because he only used it once. Nevertheless, 5-dimensions including time led to exact calculations that became a written part of theoretical physics. In what follows, there is therefore no basis for discarding Einstein's original 5-dimensional vision and its corresponding geometry.

In the exterior case, the universe 3-hypersurface is conceived as an idealized, although useful, approximation of isotropy and homogeneity by virtue of two parameters, the

mass density and pressure of a perfect cosmic fluid in the right member of Einstein's equation. Accordingly, the universe 3-hypersurface is acted upon by cosmic gravity working along radial cosmic geodesics determining Newton's gravitational potential $-Gm/R$ on the 3-hypersurface, m being the negative mass of the primeval black hole. R is the increasing radius of the universe 3-hypersurface,

Acted upon by *cosmic gravitation*, the universe 3-hypersurface expands accordingly. Astronomers and cosmologists imagine therefore the universe as if they were located at some distance from it, which is the essence of the exterior case described by the R-W metric reflecting Einstein's method used for his first universe **[1-B]**.

Although based on equations providing acceptable average values at high scale, the image of a continuous perfect fluid with pressure by definition does not detail discontinuous and complex objects such as systems of stars and planets. In relation with the big bang image, still with us in the present universe, obtaining more than rough average values is hardly viable for galaxy clusters contrasting with organized systems of stars such as anisotropic centered-spherical spiral galaxies. Moreover, Einstein's embedding a 3-dimensional universe-hypersurface in a 4-dimensional Euclidean universe (5-dimensional when including time) may look like a thought experiment when not realizing that Einstein's vision became highly physical when his universe radius became time-dependent a few years later (Alexander Friedmann, 1922).

III. Generalities

1..The structure $Y = 2Gm/rc^2$ in Newton's potential $-Gm/r$ plays a crucial role in gravitation theory because exact in Newton's theory, as well as in the static case of general relativity (see below). This Newton potential seems underestimated since retrieved in a weak field approximation from Einstein's field equation of gravitation. However, this approximation only applies to A in Eq. (1) since leading to $A \approx 1 + Y$, instead of $A = 1/(1 - Y)$ for Y considered small **[1-B]**. This approximation is corrected by the exact Schwarzschild solution maintaining $B = 1 - Y$ in the static case, which defines black hole solutions through $B = 1 - Y$, the R -dependence of A defining the expansion.

2. A massive object such as a star is expected to expand, or contract, according to a metric including a time-dependent radius. We might refer to that as "forced expansion, or contraction", possibly occurring with the R-W metric not including a source of gravitation, issue related to dark energy (similar problem with dark matter in galaxies).

3. As in the solar system, a static gravitational field deals with various motions of massive matter, so that the word "static" refers to a static gravitational field, physical concept unrelated to eventual expansion of matter. Not omitting the central black hole present in static (spherical) gravitational configurations, as in Fig. A, the universe can expand in relation with a primeval black hole located very close to the previous big bang place in Fig. A. Looking at this, it seems hard to avoid the negativity of black hole mass inducing repulsive gravitation provoking the formation of a first universe. Moreover, the gravitational force is here typically central, meaning exclusively distance-dependent, causing the radial expansion of the universe in accordance with a static, centered-spherical field configuration leaving no room for *magneto-gravitation* **[5]**.

IV. Exterior Vacuum Case according to Hamilton-Jacobi Formulation

1. Parenthesis

What follows does not recall the Lagrangian derivation of Jacobi's equations and the Hamiltonian formalism not really used here, except next trivial equation (2). In this view, the Hamilton-Jacobi formalism mainly reduces to the Jacobi equations providing a sufficient basis in the presently simple situation.

2. The Hamilton-Jacobi Formulation

In cosmology, the flat Euclidean 4-space plus time in Fig. A corresponds to the exterior case describing the geometry inside an expanding universe 3-hypersurface, the internal geometry of this universe constituting the only object of interest in cosmology. In contrast, the Hamilton-Jacobi formalism [6, 7] exclusively describes the motion of any object of mass M in a continuum, without looking inside this object.

The way general relativity is commonly developed, its applications do not include the external motion of a universe, particularly not in a flat continuum. Quite the opposite, the Hamilton-Jacobi formulation visualizes the motion of this universe in a flat continuum where all first derivatives of the metric vanish. General relativity might therefore work in conjunction with the Hamilton-Jacobi formulation. In other words, we go ahead with this.

The Hamilton-Jacobi theory also includes the Hamilton-Jacobi equation detailed in the Preface of Vinti's book [8]: *...The Hamilton-Jacobi equation, which in modern physics provided the transition to wave mechanics, is now seen as the starting point for the Vinti spheroidal method for satellite orbits and ballistic trajectories...* Visualizing the Hamilton-Jacobi formulation in 4-dimensional special relativity, the (constant) Hamiltonian H of the universe reads

$$H = Mc^2/(1-v^2/c^2)^{1/2} - GmM/r = k \quad (2)$$

where k is a constant, M being the constant rest mass of the universe with its vacuum energy, by virtue of energy conservation, The mass m is the negative mass of a primeval black hole, black holes being included in all Schwarzschild (static) solutions, which is extended here to radial expansion. We assume that this primeval black hole originated our universe through repulsive gravitation implied by the negativity of black hole mass. Moreover, a vanishing initial radius before expansion would imply infinite potential energy opposing a necessarily finite rest mass M of the universe, according to k being constant in Eq. (2). To obviate this difficulty, we consider next scenario detailing four distinct phases.

The first phase is the apparition of an anisotropic, centered-spherical, black hole of negative mass, inevitably related to Dirac's negative energy solutions.

The second phase, introducing a first big bang, describes the ejection, from the primeval black hole, of a centered-spherical universe with positive mass. This first universe corresponds to Lemaître's *primeval atom* (see further).

The third phase presents a second big bang occurring during the transition from the anisotropic, centered-spherical, universe to a second isotropic universe through symmetry breaking. The fourth phase marks the start of universe expansion at the time zero.

Summarizing, this proposed context defines the time zero as the start of universe expansion occurring after formation of two successive universes, according to

$$-GMm/r = -GMm/X \quad (3)$$

($t = 0$), where X is the initial finite radius of the second (isotropic) universe.

In line with Eq. (2), Jacobi's writings describing the motion of a massive body (1834-1843), introduced the action S whose infinitesimal variation δS reads

$$\delta S = \mathbf{p} \cdot \delta \mathbf{x} - W \delta t \quad (4)$$

where W is the energy, \mathbf{p} being the 3-momentum. Eq. (4) is consistent with

$$p_x = \partial S / \partial x ; \quad W = -\partial S / \partial t \quad (5)$$

according to

$$S = \mathbf{p} \cdot \mathbf{x} - Wt \quad (6)$$

with the particular wave-solution [6]

$$\psi = \exp[iS/\hbar] \quad (7) ; \quad (\hbar \equiv h/2\pi , \quad i = \sqrt{-1})$$

defining de Broglie's wave ψ , interpreted here as universe wave-function. In the exterior case, the universe is essentially a massive body whose radial motion implies its vanishing angular momentum, so that (7) reduces to

$$\psi = \exp[i/\hbar(pR - Wt)] \quad (8)$$

where R is the universe radius. In addition, the action S is an invariant in

$$S \equiv \mathbf{p} \cdot \mathbf{x} - Wt = p_{\mu}x_{\mu} + (iW/c)(ict) \quad (9)$$

($x_4 = ict$, Greek indices refer to space). The concept of universe wave-function is not new [9], as theorists felt free to enlarge Bohr's correspondence beyond microphysics according to quantum reality coexisting with classical physics everywhere, which is simpler than limiting the application of Bohr's correspondence principle. All this, not counting the relation between primeval black hole mass and Dirac's negative energy solutions.

A bit surprisingly, Dirac's equation for the electron looks like the only theoretical basis sustaining the sudden apparition of negative black hole mass in a flat, 5-dimensional vacuum spacetime, which eliminates spinor coupling with gravitation, commonly seen as the major difficulty.

3. Comments

1. Probability densities follow the radial lines of cosmic gravitation causing the expansion of the universe. Separations between centers of mass of systems of stars such as galaxies maintain their radial alignment along the lines of cosmic gravity related to fixed stars. Voids between galaxy clusters only enlarge according to the expansion. In other words, after the double big bang the universe kept the general configuration it had at the time zero, the CMB (cosmic microwave background) being distributed quite simultaneously.

2. The instability of the first universe, of positive mass produced by the primeval black hole, is not detailed here since requiring some elements of galactic gravitation (Part 2). It is, however, conceivable that this first universe had the same symmetry as the primeval black hole. Afterward, the change of geometry, from centered-spherical anisotropy to spherical isotropy, provoked a chaos caused by this symmetry breaking, which added to the first explosive emission of hot matter from the black hole. Both (successive) phenomena constitute a double big bang whose image, enlarged through radial expansion, should correspond to that of the large-scale structure of the present universe. As introduced above, this hypothesis seems consistent with observed voids and other irregularities in astronomical pictures, the North-South blue haze in a picture of the 2MASS Project giving the impression of a possible footprint of this symmetry breaking ?

3. Regarding black holes, the reported story (lost references) was that Gold, Bondi and Hoyle founded their *Steady State Theory* (1948) on observations of violent explosions from the black hole Sagittarius A (roughly 3 or 4 million solar masses), located at the center of our galaxy. These observations led to comments such as *...violent events do seem to be occurring in the nuclei of many galaxies, so galactic nuclei seem like natural candidates for the location of continuous creation* [2]. But black holes at centers of spiral galaxies do not have negative mass-energy like the primeval black hole having here originated the big bang. Moreover, non expanding galaxies seem to emit as much mass-energy as that they absorb (approximation), in possible relation with the

hypothesis of gravitational radiations of positive and negative energies [10] on which more work needs to be done. Nevertheless, galactic black holes look like small big bangs according to Prigogine's comments on French speaking TV (around 2003 ? - not reported on Internet). As a first conclusion, we will only understand black holes and their *mystery* when clarifying their field structure in a (centered-spherical) galactic configuration.

4. According to Luminet (Internet), Lemaître's concept of a primeval atom, synonymous to *primitive atom*, came out before 1930. Two years later, Lemaître recalled the old concept of primeval nebula introduced by Kant and Laplace [11], who detailed how a diffused primeval nebula (not yet Lemaître's atom) filled the whole space and progressively condensed into partial nebulae, finally producing stars. Lemaître also developed some themes related to the cosmology of his time, for example mentioning the big crunch in which he did not believe, which is casually confirmed by the simple Hamilton-Jacobi model detailed above. According to this model, a stabilization of the universe, after exhaustion of (positive) potential energy in Eq. (2), will only occur if the primeval black hole mass is constant, which works according to energy conservation (see below). Somewhat detailing this, the primeval black hole constitutes an initial condition, apparently contradicting any conservation principle. However, such principles only make sense within an existing spacetime, which here appears during, or after, the formation of the first spacetime. Lemaître also underlined the great importance of the missing mass problem. Moreover, Dirac's work (see above) confirms Lemaitre's vision following which the immense universe is based on quantum microphysics, reason why he replaced the Kant-Laplace word *nebula* by *atom*, conceptually covering atomic physics of quantum essence, independently of the reductionist concept of atomic smallness.

5. Lemaître's concept of primeval atom looks therefore like a key opening a door to major theoretical issues he related to quantum physics. Confirming this, a crucial paper appeared in 1978 [12], detailing the big bang through the apparition of a quantum fluctuation, whose energy exactly compensates the negative gravitational energy by virtue of energy conservation, respecting therefore causality according to the authors (important). The basic concept of this 1978 paper somewhat differs from the present paper presenting a supposedly giant black hole as initial condition. We then assume that positive energy was extracted from a fraction of the primeval negative mass, slightly raising black hole mass negativity, quite simultaneously with the emergence of spacetime with positive vacuum energy, process without which space and time could not surface out of nothingness (spacetime vacuum should have a cost). Summarizing, energy conservation applies from the time zero, or the beginning of the formation of the first universe. Afterward, the negative black hole mass remains constant forever, so that the universe will reach a maximum size after total exhaustion of the positive potential energy.

Evidently, all this does not minimize Einstein's equation, quite far from having said its last word.

V. Origin of the R-W Metric

Consider a spherical 3-dimensional hypersurface defining the universe embedded in a 4-dimensional Euclidean continuum **[1-B]** according to

$$x^\beta \cdot x^\beta + x^4 \cdot x^4 = a^2 \quad (10)$$

with

$$dl^2 = dx^\beta \cdot dx^\beta + dx^4 \cdot dx^4 \quad (11)$$

($\beta = 1, 2, 3$ - Greek indices refer to space), where a is Einstein's constant curvature radius, dl^2 being the squared infinitesimal 4-dimensional distance between two neighboring points. Differentiating (10) gives

$$x^\beta \cdot dx^\beta + x^4 \cdot dx^4 = 0 \quad (12)$$

or,

$$dx^4 = - (1/x^4) x^\beta dx^\beta \quad (13)$$

implying

$$(dx^4)^2 = (1/x^4)^2 \cdot x^\alpha x^\beta dx^\alpha dx^\beta \quad (14)$$

Eq. (10) gives

$$(x^4)^2 = a^2 - x^\beta x^\beta \quad (15)$$

So that Eq. (14) becomes

$$(dx^4)^2 = [1/(a^2 - x^\gamma x^\gamma)] x^\alpha x^\beta dx^\alpha dx^\beta \quad (16)$$

Eliminating x^4 in Eq, (11) then gives

$$(dl)^2 = \{\bar{\delta}_{\alpha\beta} + [x^\alpha x^\beta / (a^2 - x^\gamma x^\gamma)]\} dx^\alpha dx^\beta \quad (17)$$

where $\bar{\delta}_{\alpha\beta}$ is the Kronecker symbol.

Because important, we recall Einstein's footnote referring to Eqs. (10) and (11) **[1-B]**: *The aid of a fourth space dimension has naturally no significance except that of a mathematical artifice.* Although agreeing with Einstein's approach, we do not go as far as ratifying his prudence undervaluing a correct mathematical operation by calling it an

artifice. Quite the opposite, embedding the 3-dimensional universe in a Euclidean 4-space refers to the work of a theorist describing the exterior case by drawing and calculating a universe on a piece of paper. Doing this, the researcher imagines he is in a 4-dimensional Euclidean space, which will become physical if his equations prove to be valid. This context was that of Einstein's mind, enabling him to visualize the 3-dimensional isotropic curved hypersurface of his first static universe, from which the Robertson-Walker metric derives, which is now detailed. Differentiating the definition

$$x^\beta x^\beta \equiv r^2 \quad (18)$$

gives the key relation

$$x^\beta dx^\beta = r dr \quad (19)$$

so that writing $dx^\beta dx^\beta$ in spherical coordinates

$$dx^\beta dx^\beta = dr^2 + r^2(d\theta^2 + \sin^2\theta.d\phi^2) \quad (20)$$

puts Eq. (17) in the form

$$(dl)^2 = dr^2 + r^2 dr^2/(a^2 - r^2) + r^2(d\theta^2 + \sin^2\theta.d\phi^2) \quad (21), \text{ or}$$

$$(dl)^2 = a^2 dr^2/(a^2 - r^2) + r^2(d\theta^2 + \sin^2\theta.d\phi^2) = dr^2/(1 - r^2/a^2) + r^2 d\Omega^2 \quad (22)$$

where $d\Omega^2$ is the usual notation for $d\theta^2 + \sin^2\theta.d\phi^2$.

Through the change of variables $r^* = r/a$, (22) leads to

$$(dl)^2 = a^2 dr^{*2}/(1 - r^{*2}) + r^{*2}.d\Omega^2 \quad (23)$$

here written in standard form [13], where r^* is dimensionless and a is Einstein's universe radius. The right member of Eq. (23) is the 3-dimensional part of the usual R-W metric. Since momentarily not multiplied by a^2 , the dimensionless term $r^{*2}d\Omega^2$ in this space metric in standard form is somewhat devoid of physical meaning, because the vanishing of any metric in standard form is not generally applicable to the calculation of the speed of light. However, the conversion of (22) to (23) corresponds to a coordinate transformation [2] not affecting the field equations by virtue of general covariance, the only motive being a simpler derivation of the field equations.

VI. A Variant of the R-W metric

Discarding inapplicable maximal symmetry [2], the following expression of B from Eq. (1) includes the negative source-mass m of the primeval black hole, so that

$$B = (1 - 2Gm/Rc^2) \quad (24)$$

which is the usual Schwarzschild black hole solution in the static case, based on Newton's potential $-Gm/r$, with $r = R$ on the equipotential of the universe-3-hypersurface. According to the above coordinate transformation $r = a.r^*$, where a was Einstein's constant universe radius of his first static universe, we replace r by $R.r^*$, in conformity with what was done above to retrieve the 3-dimensional part of the R-W metric in the form of Eq. (23). Not using the standard form and treating the universe radius R as a time-dependent variable, the complete R-W metric reads

$$ds^2 = c^2[1 - 2Gm/Rc^2]dt^2 - R^2[dr^{*2}/(1 - r^{*2}) + r^{*2}d\Omega^2] \quad (25)$$

Suppressing the unnecessary asterisks then gives

$$ds^2 = c^2[1 - 2Gm/Rc^2]dt^2 - R^2[dr^2/(1 - r^2) + r^2d\Omega^2] \quad (26)$$

where r is dimensionless.

REFERENCES

[1-A] G. Lemaître: *Un univers homogène de masse constante et de rayon croissant rendant compte de la vitesse radiale des nébuleuses extra-galactiques* Comptes Rendus Soc. Scient. Bruxelles, 49, April 25, 1927), p.11.

G. Lemaître: *L'Univers en Expansion*, Annales Soc. Scient. Bruxelles, Série A, pp. 51-85.

[1-B] A. Einstein: *The Meaning of Relativity* (Princeton University Press, 5th ed., NJ 1955), pp. 86, 90, 100, 102, 104, 109, 111.

[2] S. Weinberg: *Gravitation and Cosmology* (Wiley, NY 1972), pp. 87, 177, 300, 302, 412, 413, 460, 471, 491, 613.

[3] M.-A. Tonnelat: *Les Théories Unitaires de l'Électromagnétisme et de la Gravitation* (Gauthier-Villars, Paris 1965), pp. viii, ix, xix.

[4] J. Chauveheid: *A Proposal for a Quadratic Electromagnetic Coupling Based on the Underlying Philosophy of Einstein-Maxwell Theory*, Phys. Essays, vol.10, Nr 3, p. 474 (1997).

[5] R.M. Wald: *General Relativity* (Chicago University Press, 1984), p. 78.

[6] W. Yourgrau and S. Mandelstam: *Variational Principles in Dynamics and Quantum Theory* (Dover, NY 1979), p. 123.

[7] J. Chauveheid and F.X. Vacanti: *Schrödinger and the Hamilton-Jacobi Equation*. Phys. Essays, Vol. 15, Nr 1, p. 5 (2002).

{8} John Vinti, in

https://books.google.co.cr/books/about/Orbital_and_Celestial_Mechanics.html?id=-dXzdYHvPgMC&hl=es

<http://adsabs.harvard.edu/full/1961AJ.....66..514V>

[9] J. Hartle and S. Hawking: *Wave function of the Universe*, Phys. Review, D28, (12): 2960 (1983).

[10] F.X. Vacanti and J. Chauveheid: *Is Radiation of Negative Energy Possible?*. Physics Essays, Vol. 15, Nr 4, p. 387 (2002).

[11] G. Lemaître: *L'Expansion de l'Espace* (Ets. Fr. Ceuterick, Louvain, 1932), pp. 5. 23.

Georges Lemaître in <https://www.uclouvain.be/en-316446.html>

Note: Significant Lemaître's papers, starting with the rare original 1927 paper, are mailed free of charge by Mrs. Moens of the *Lemaître Foundation* (Louvain, Belgium - email in this web page).

[12] R. Brout, F. Englert, E. Gunzig :*The Causal Universe*, Faculté des Sciences, Brussels University 1978/

[13] Y. Choquet-Bruhat: *General Relativity and the Einstein Equations* (Oxford University Press, NY 2009), chapter 5, section 4, Eq. 4.26.